New pupil masks for high-contrast imaging

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Objective

To design high-throughput pupil masks that provide high-contrast in the image plane at close inner working angles.

Outline

- Apodization
- Prolate Spheroidal Wave Function
- Concentric Ring Masks
- Spiderweb Masks
- Starshape Masks
- Asymmetric Masks
- Summary

Apodization

The image-plane *electric field* produced by an on-axis plane wave and an apodized aperture defined by an *apodization function* $A(r)$ is given by

$$
\begin{aligned}
E(\rho) &= \int_0^{1/2} \int_0^{2\pi} e^{-2\pi i r \rho \cos(\theta - \phi)} A(r) r \, d\theta \, dr, \\
&= 2\pi \int_0^{1/2} J_0(2\pi r \rho) A(r) r \, dr,
\end{aligned}
$$

where J_0 denotes the 0-th order Bessel function of the first kind.

The unitless pupil-plane "length" r is given as a multiple of the aperture D.

The unitless image-plane "length" ρ is given as a multiple of focal-length times wavelength over aperture $(f\lambda/D)$ or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D.

The *point spread function* (psf) is the square of the electric field.

Performance Metrics

Inner Working Angle

$$\rho_{\text{iwa}}$$

Contrast:

$$E^2(\rho)/E^2(0)$$

Airy Throughput:

$$\frac{\int_0^{\rho_{\text{iwa}}} E^2(\rho) 2\pi \rho d\rho}{(\pi(1/2)^2)} = 8 \int_0^{\rho_{\text{iwa}}} E^2(\rho) \rho d\rho.$$

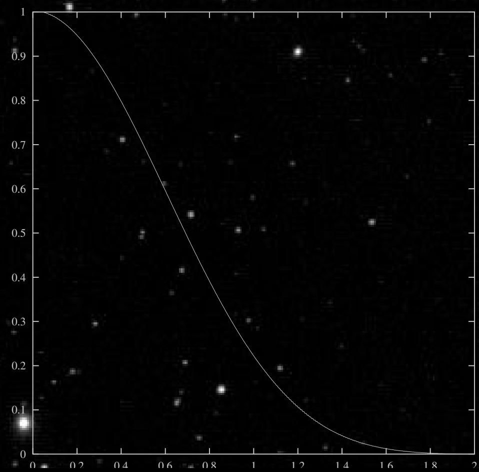
Prolate Spheroidal Wave Function

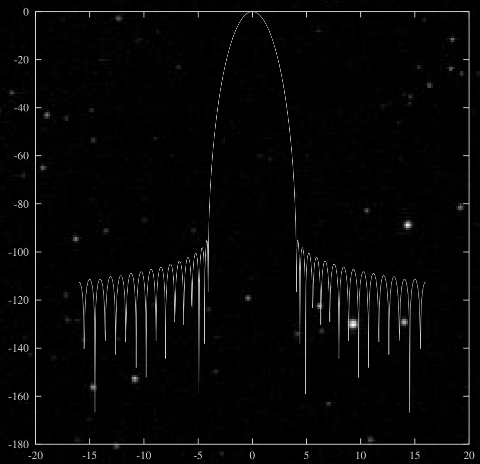
Slepian's *generalized prolate spheroidal wave function* is defined as the solution to the following optimization problem:

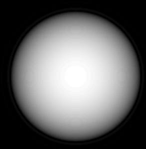
$$\text{maximize } \frac{\int_0^{\rho_{\text{iwa}}} E(\rho)^2 \rho d\rho}{\int_0^{\infty} E(\rho)^2 \rho d\rho}.$$

Choosing $\rho_{\text{iwa}} = 4$ provides 10^{-10} contrast for $\rho \geq \rho_{\text{iwa}}$.

$$\rho_{\mathrm{iwa}} = 4$$
$$\rho_{\mathrm{owa}} = \infty$$
Airy Thruput = 9%.







Computational Tractibility

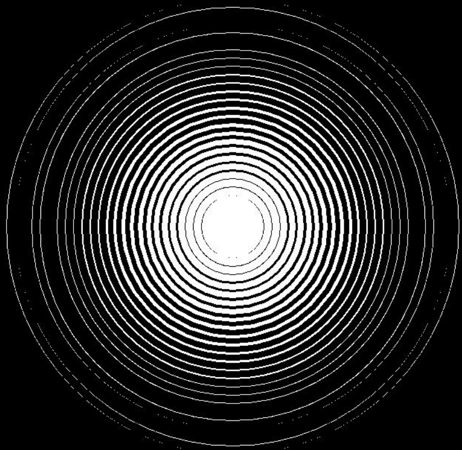
It's important to avoid $E(\rho)^2$ in the model:

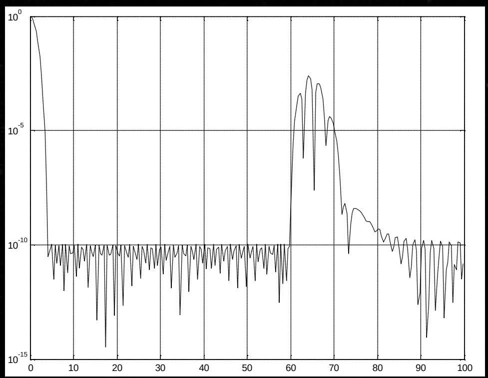
$$\begin{aligned}
&\text{maximize} && \int_0^{1/2} A(r) 2\pi r dr \\
&\text{subject to} && -10^{-5}E(0) \leq E(\rho) \leq 10^{-5}E(0), && \rho_{\text{iwa}} \leq \rho \leq \rho_{\text{owa}}, \\
& && 0 \leq A(r) \leq 1, && 0 \leq r \leq 1/2,
\end{aligned}$$

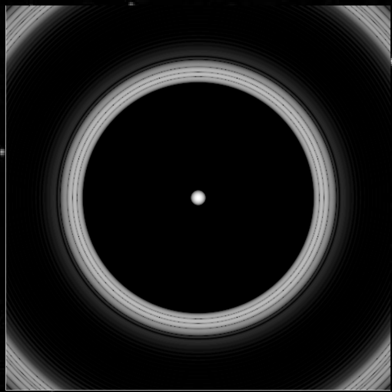
The solution is zero-one valued. That is, it is a concentric ring mask. The number of rings tends to infinity as $\rho_{\text{owa}} \rightarrow \infty$.

Concentric Ring Mask

$\rho_{\text{iwa}} = 4$
$\rho_{\text{owa}} = 60$
Airy Thruput $= 9\%$.







Spiderweb Masks

Concentric rings can be supported by "pie-shaped" spiders:

$$S_n = \left\{ (r,\theta) : \; 0 \le r \le 1/2, \; \frac{2\pi n}{N} + \frac{\alpha}{2} \le \theta \le \frac{2\pi(n+1)}{N} - \frac{\alpha}{2} \right\}.$$

Overlaying these spider vanes on an apodization $A(r)$ yields the following electric field:

$$\begin{aligned} E(\rho,\phi) \;\approx\; & 2\pi \int_0^{1/2} J_0(2\pi r\rho)(1 - N\alpha/(2\pi))A(r)r\,dr \\ & -4 \int_0^{1/2} J_N(2\pi r\rho)\cos(N(\phi - \pi/2))\sin(N\frac{\alpha}{2})A(r)r\,dr. \end{aligned}$$

The impact of the spiders is two-fold:

- The original electric field is reduced by $1 - N\alpha/(2\pi)$ which is just the fraction of the aperture that remains uncovered by the spiders.
- There is an additional term. It produces diffraction spikes.

Diffraction Spikes are Bessel Modulated

57 Cyg
with Schuler 4.5 nm H-alpha filter

Photo courtesy of Brian Sledz

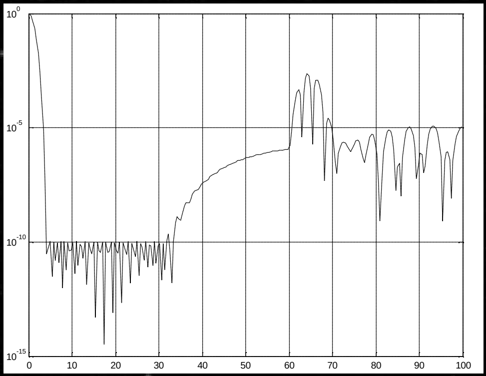
Diffraction Spikes

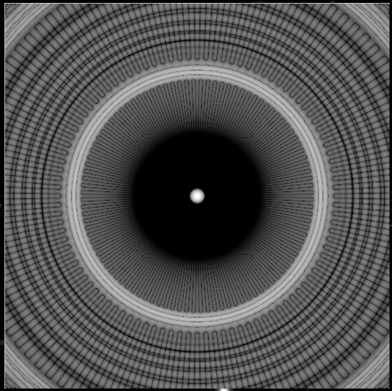
A small number of spiders implies bright diffraction spikes that destroy the required contrast.

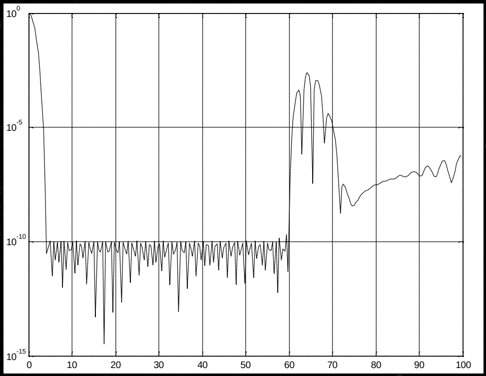
More spiders means more spikes. Sounds bad...but, more spiders also means that the spikes move out away from $\rho = 0$. With enough spiders, the high contrast zone can be preserved.

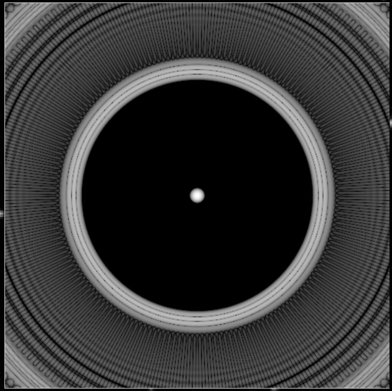
The next page shows the psf one gets using 100 and 180 spiders.

PSF's for 100 and 180 Spider Vanes









Starshape Masks

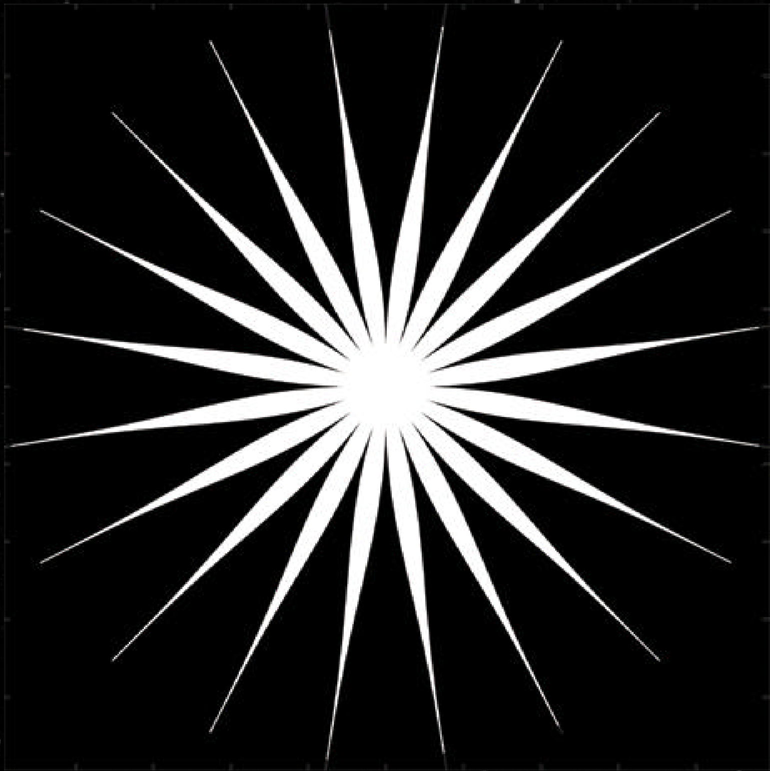
Dispense with the concentric rings and let the spider width α vary with radius r so as to approximate any given apodization $A(r)$:

$$\alpha(r) = \frac{2\pi}{N}\left(1 - A(r)\right).$$

Here, N denotes the number of vanes in the mask.

Again, there are diffraction spikes that move out as N increases. A value of N of about 150 gives a sizeable high-contrast discovery zone.

A 20-Point Starshape Mask



Which Apodization?

There is a starshape mask associated with every apodization.

The prolate spheroidal is okay but perhaps not the best.

The "optimal" one turns out to be zero-one valued which yields a starshape mask that is exactly a system of concentric rings.

Motivated by a Gaussian apodization, we introduce smoothness constraints:

$$(\log A)' \leq 0, \qquad (\log A)'' \leq 0.$$

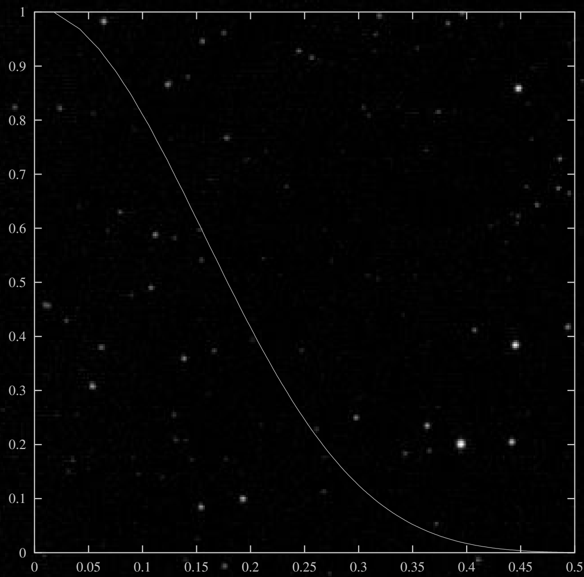
The resulting optimization problem is:

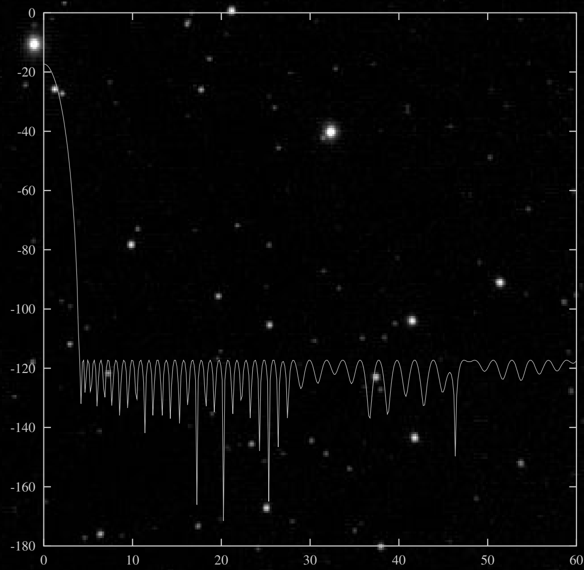
$$\begin{array}{llcll} \text{maximize} & \int_0^{1/2} A(r) 2\pi r dr & & & \\ \text{subject to} & -10^{-5}E(0) \leq & E(\rho) & \leq 10^{-5}E(0), & \rho_{\text{iwa}} \leq \rho \leq \rho_{\text{owa}}, \\ & 0 \leq & A(r) & \leq 1, & 0 \leq r \leq 1/2, \\ & & A'(r) & \leq 0, & 0 \leq r \leq 1/2, \\ & & A(r)A''(r) & \leq A'(r)^2, & 0 \leq r \leq 1/2. \end{array}$$

Optimal Smooth Apodization

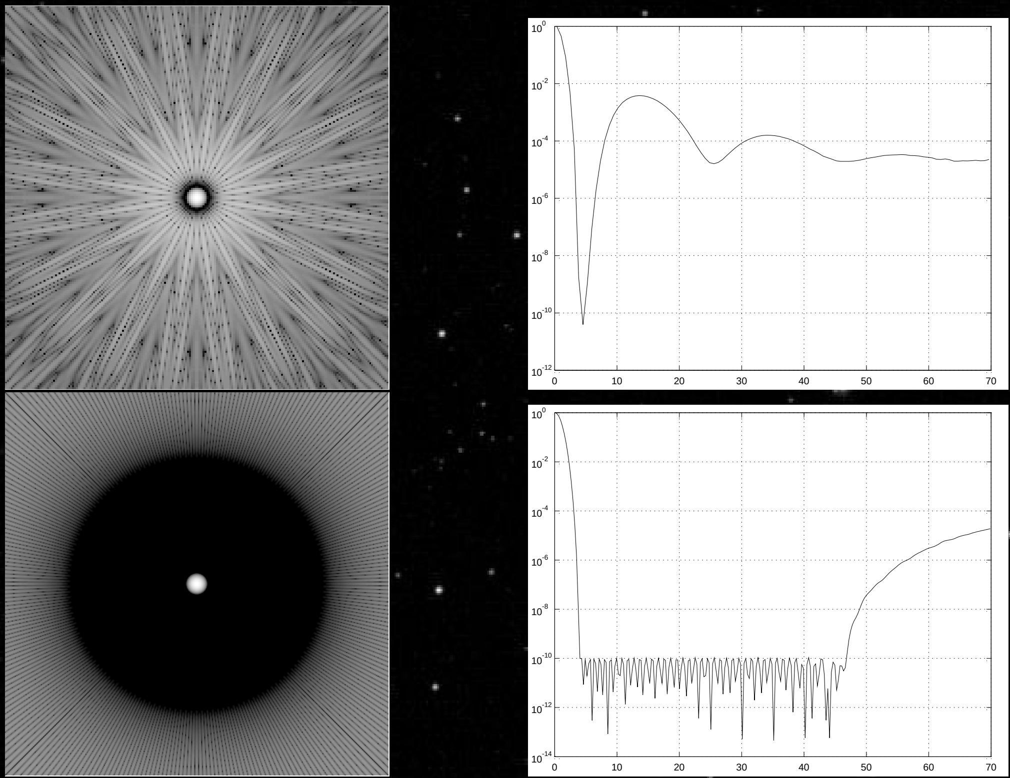
$\rho_{\text{iwa}} = 4$, $\rho_{\text{owa}} = 60$,

Airy Throughput $= 9\%$.





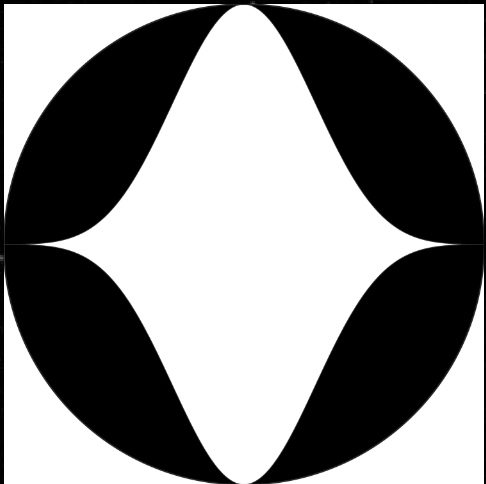
PSF's for 20 and 150 Point Stars

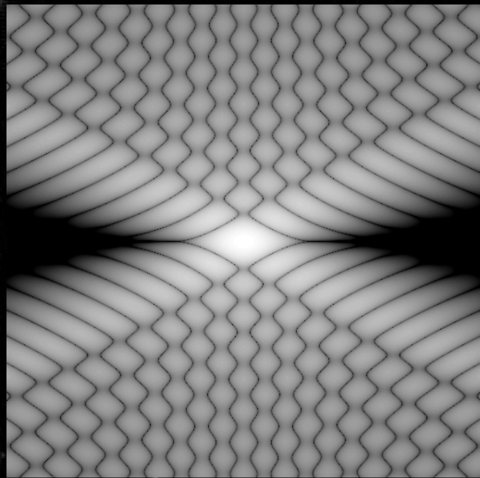


Asymmetric Masks (for comparison)

Prolate Spheroidal Mask

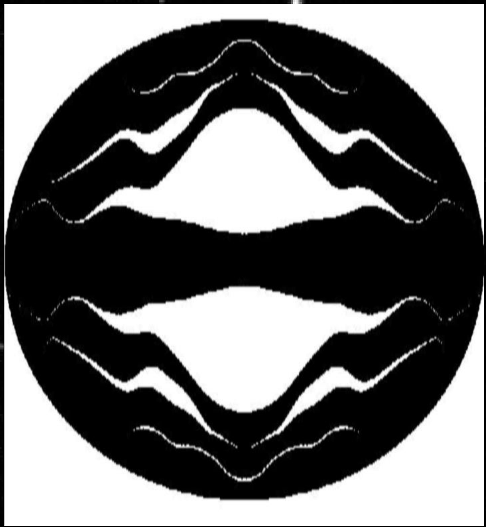
$\rho_{\text{iwa}} = 4$
$\rho_{\text{owa}} = \infty$
Airy Thruput $= 43\%$

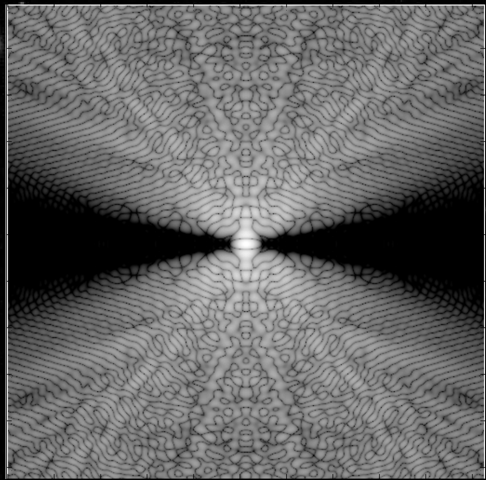




Multiple Openning Mask

$\rho_{\text{iwa}} = 4$
$\rho_{\text{owa}} = 100$
Airy Thruput $= 12\%$





Summary

	ρ_{iwa}	ρ_{owa}	Airy Thruput	Number of Integrations
Concentric Rings	4	60	9.37	1
Spiderweb with 10% spiders	4	60	7.59	1
Starshape	4	60	9.09	1
Asymmetric Prolate	4	∞	42.66	∞?
Asymmetric Multiple	4	100	11.85	10

- The asymmetric masks have the highest throughput but require many integrations to investigate all angles around the star. Hence, for discovery, they are the worst but if the orientation is known then they are the best.
- For discovery, the starshape mask is the best.
- The choice between starshape and spiderweb masks probably will be determined by manufacturing constraints.

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