Horsing Around on Saturn

Robert J. Vanderbei

Joint with J.R. Gott III and E. Belbruno

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Operations Research and Financial Engineering
Princeton University
Princeton, NJ 08544
http://www.princeton.edu/∼rvdb

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The Big Picture

- Big-bang "dust" condenses into rotating spiral galaxies.
- Within galaxies, interstellar dust condenses into stars with protoplanetary dust disks that condense further to form planetary systems.
- Planets form as rotating bodies with moons and rings in the equatorial plane.
- Jupiter, Saturn, Uranus, and Neptune all have ring systems.

Question: Why do we think planetary ring systems are unusual (transient)?

Prevailing View of Saturn

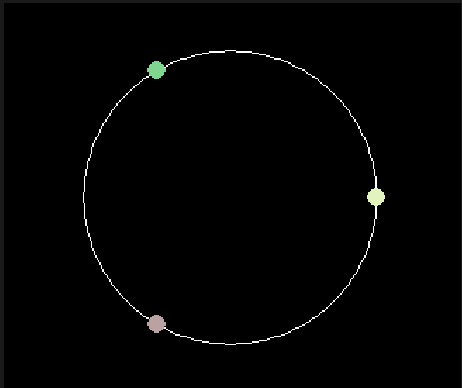
- A moon-sized object wandered inside Saturn's Roche Limit.
- The object was torn apart by tidal forces.
- The resulting array of remnant masses became dispersed into the rings.
- The system dynamics is well modeled as a turbulent flow (with little or no mention of the law of gravitation).
- The ring shape is maintained by the coralling effect of a few shepherding moons.
- Saturn's rings are relatively new and transient.

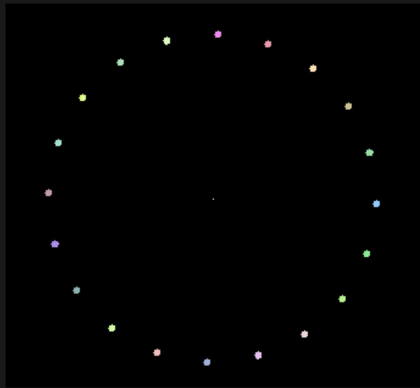
An Alternate View:

- Original dust settles into L4/L5 configurations.
- Outside Roche limit, you can get bodies at same radius.
- Saturn's rings formed with Saturn and are stable.

First Thought

Ring Systems Should be Unstable





For 3 or more bodies, the system is unstable.

NOTE: On this and subsequent pages, most graphic images are links to JAVA *applets that animate the motion. Click on 'em.*

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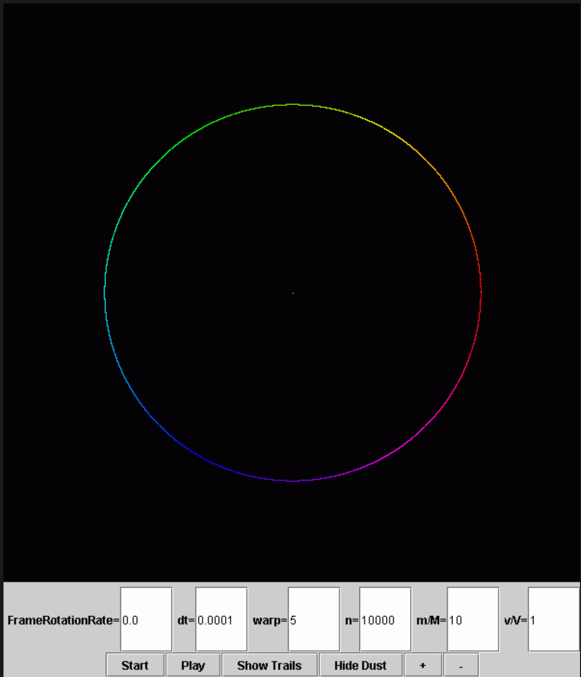
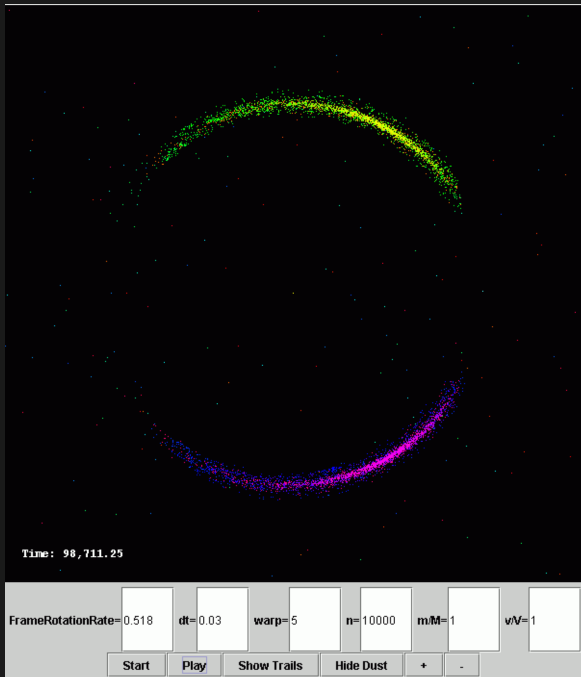
A Large Central Mass Stabilizes

Lagrange Points L4/L5 are Stable (For Modest Masses)

Trojan Asteroids

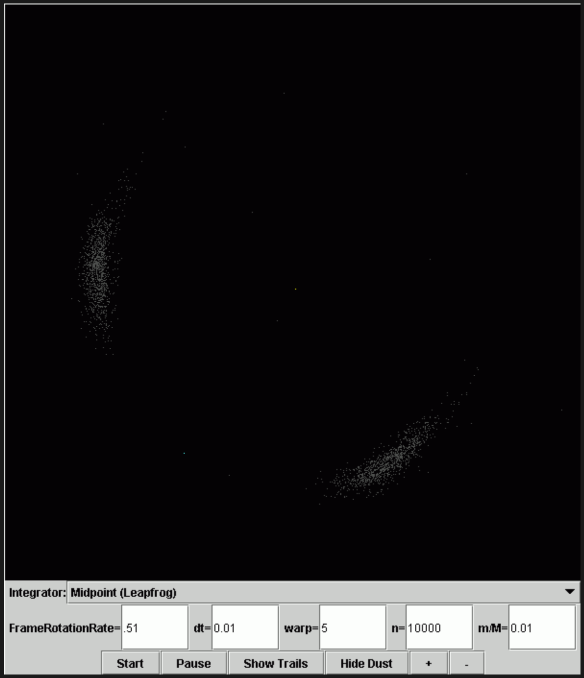




A Large Central Mass Stabilizes

Lagrange Points L4/L5 are Stable (For Modest Masses)

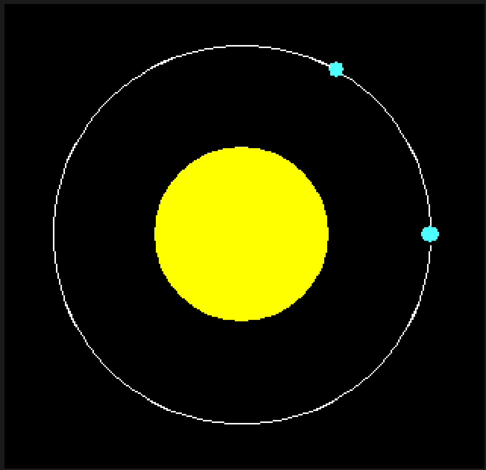
Trojan Asteroids



Two Moons of Saturn

Janus and Epimetheus

Consider Saturn and its moons Janus and Epimetheus.



Orbital radii differ by only 55km.

Let's put them at the exact same radius.

Start them in L4/L5 configuration.

$m_{\saturn}/m_{\mathsf{J}} = 2.8 \times 10^8$
$m_{\saturn}/m_{\mathsf{E}} = 1.0 \times 10^9$.

Stable if $m_1/m_2 \geq 25$ and $m_3 = 0$ (via rigorous analysis).

Stable if $m_1/\max(m_2, m_3)$ is sufficiently large (via simulation).

Stable even if eccentricity is not zero (but mass-ratio threshold is larger).

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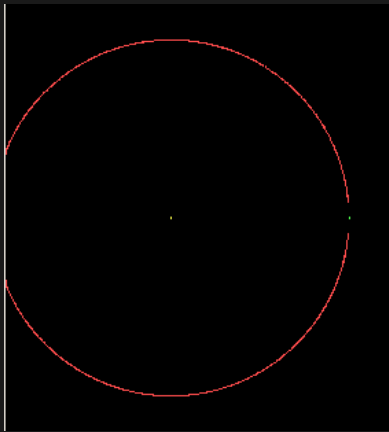
Janus and Epimetheus “Horseshoe”

They Aren’t Really in an L4/L5 Configuration

- Orbital radii that differ by 55km.
- Orbit Saturn once every 17 hours.
- Lower one orbits slightly faster.
- Every four years, the lower one overtakes the higher one.
- As it slowly creeps up from behind, their mutual gravitational pull tends to speed up the lower one and slow down the higher.

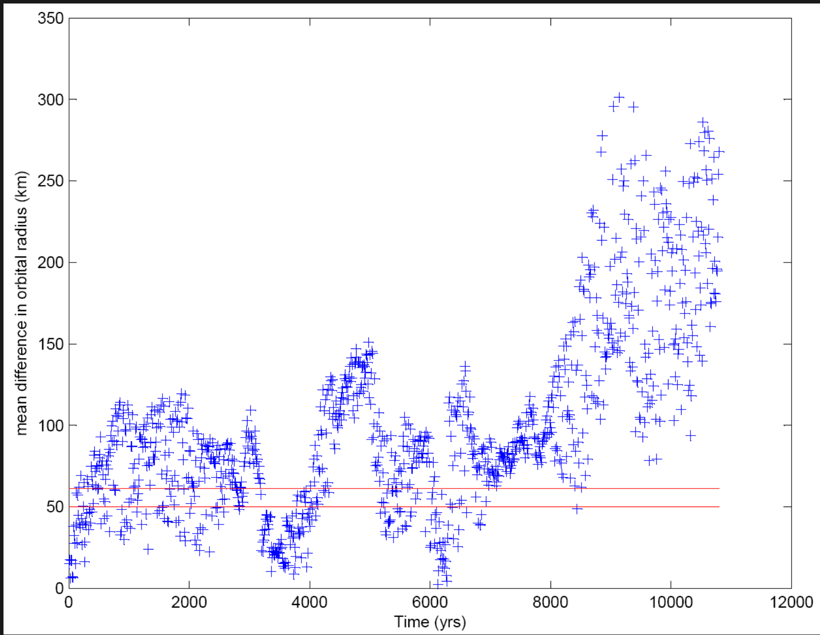


- The higher one then drops to a lower orbit and speeds up.
- The lower swings up into a higher orbit and slows down.
- Their roles have switched—fast one does not pass slow one.
- The process continues indefinitely.

How Might Such a System Form?

Perhaps the two moons formed at the essentially the same orbital radius.

Perhaps even in an L4/L5 configuration, but the masses are so small they might have formed much further apart without feeling the influence of the other.

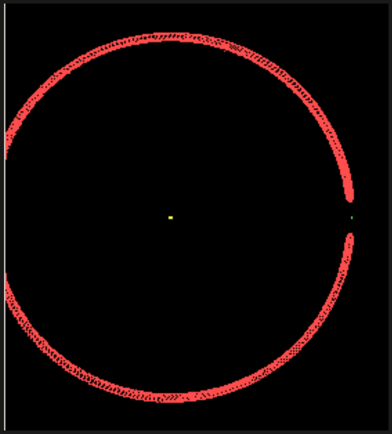


Random Δv's might then have perturbed them into the horseshoe orbit we see today.

Saturn/Janus/Epimetheus System

From Horizons Ephemeris

- Eccentricity of Janus is 0.004.
- Eccentricity of Epimetheus is 0.022.
- Inclination of Janus is 0.14 degrees.
- Inclination of Epimetheus is 0.34 degrees.
- Still, they horseshoe.
- Horseshoeing is robust.

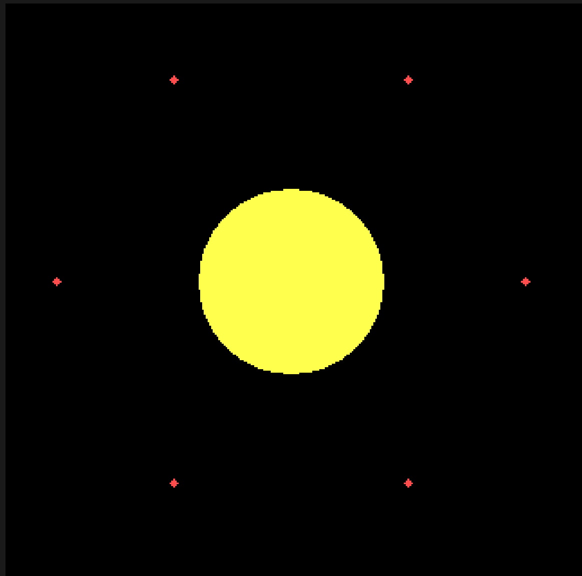


One Mass's L5 Is Another Mass's L4...

m_1 is a Saturn mass

Others are Janus masses.

Every $10^4 \Delta t$, each mass gets a random Δv of magnitude $2.5\ 10^{-6}$au/yr.

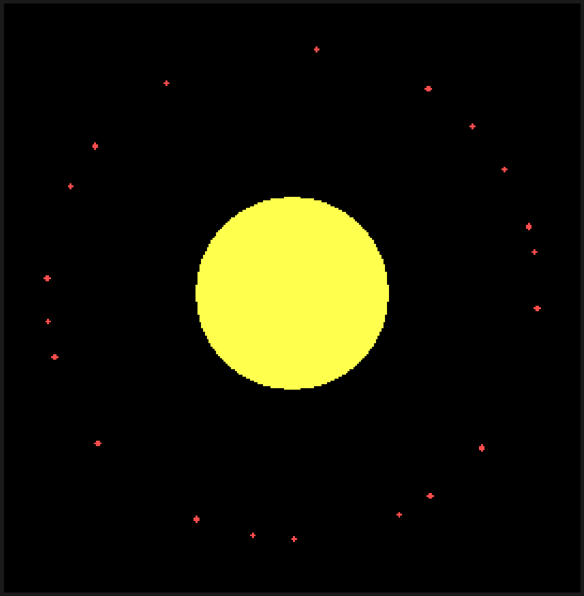


Small masses horseshoe—they neither overtake nor are overtaken.

Works with Even More Masses

Here's 20 Janus masses

Number of masses can be increased to billions provided total mass remains small.



Gravity scales nicely—a marble orbits a bowling ball every 90 minutes if it is in low-bowling-ball orbit. (Assumption: bowling ball has roughly same density as Earth.)