

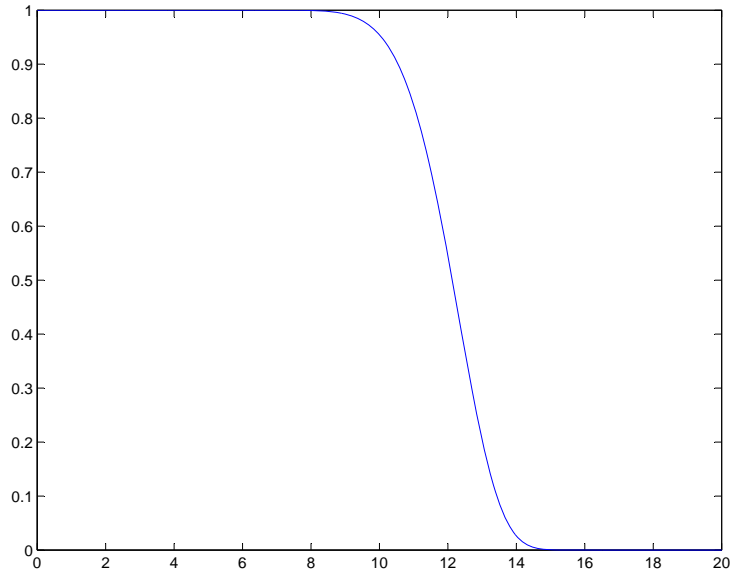
Deep Shadow Occulters

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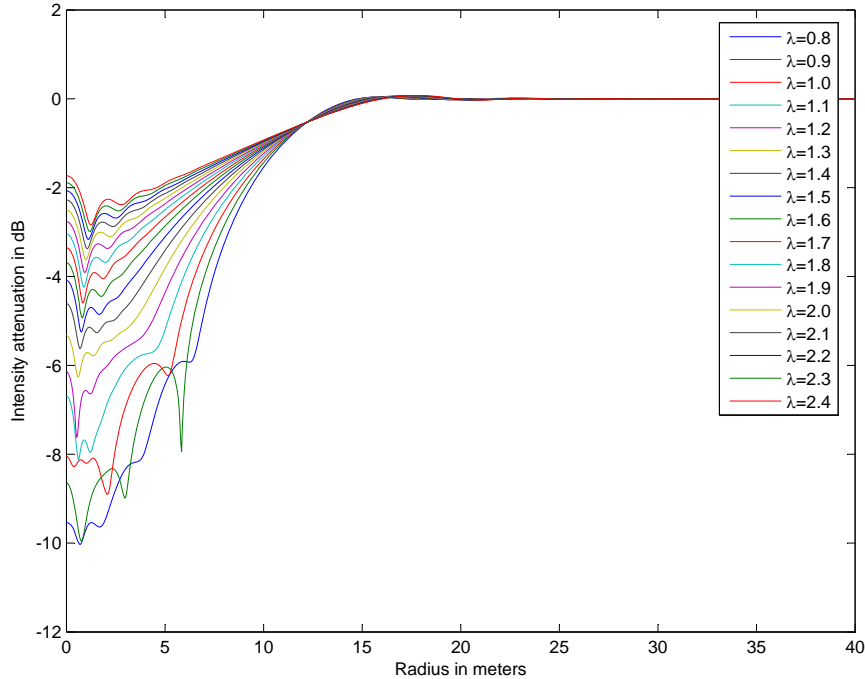
Web's Hypergaussian Radial Amplitude Profile



$$A(r) = \begin{cases} 1 & \text{for } r < a \\ e^{-((r-a)/b)^n} & \text{for } a \leq r < c \\ 0 & \text{for } c \leq r \end{cases}$$

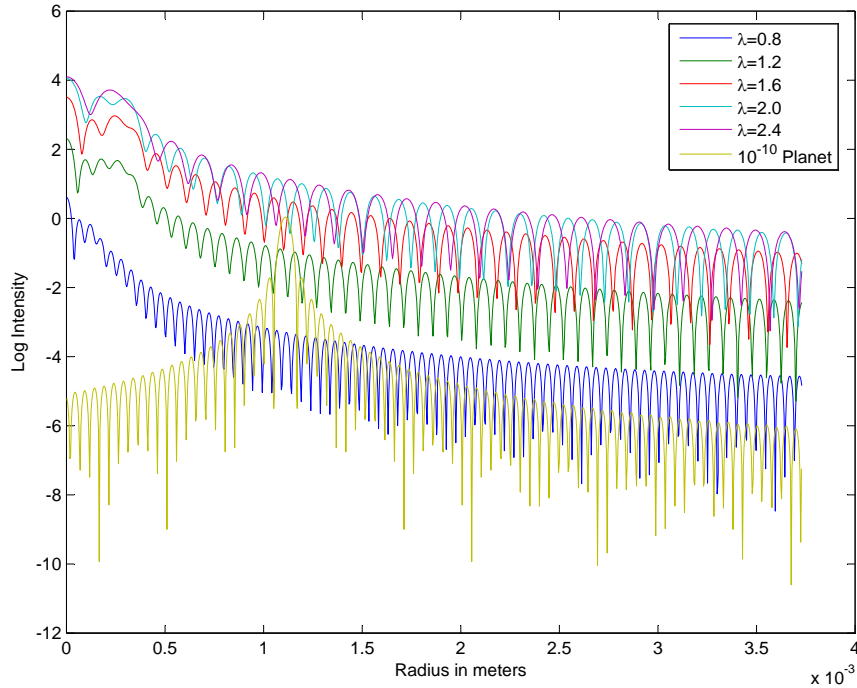
where $a = b = 6.25$, $c = 20$, $n = 6$ and distance from JWST is 18,000 km.

Performance at Several Wavelengths



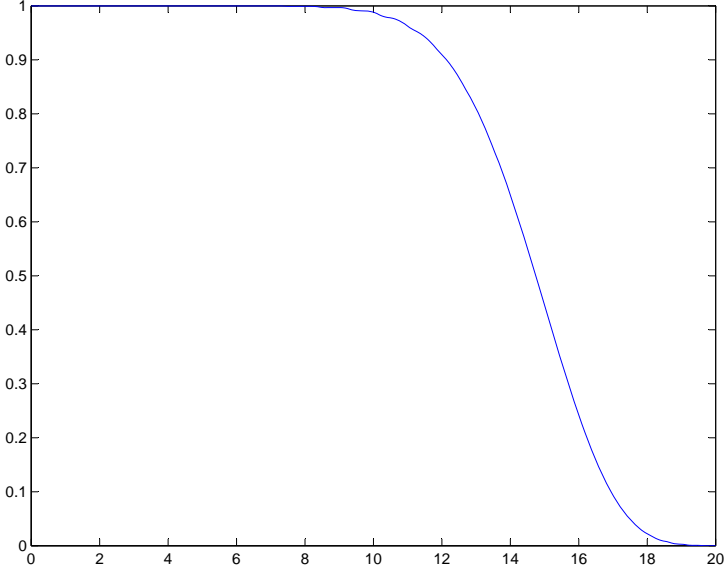
Okay at visible wavelengths, but degrades quickly as wavelength increases.

Image Plane PSFs



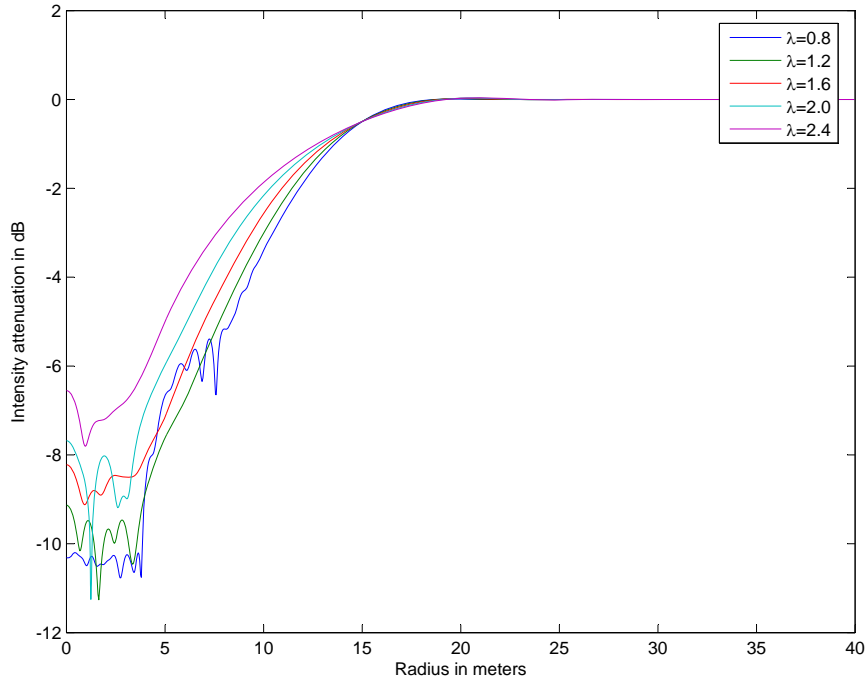
Planet at tip of flower. Planet intensity 10^{-10} times intensity of star at $\lambda = 0.8\mu$.

Optimized Radial Amplitude Profile



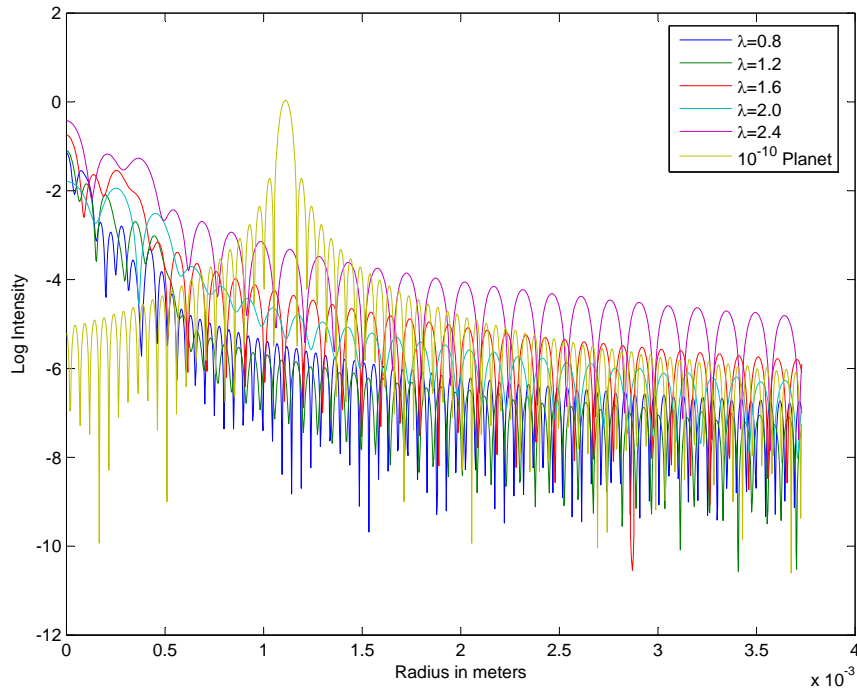
Distance from JWST: 18,000 km

Designed for Deep Shadow



$10^{-10.5}$ at $\lambda = 0.8\mu$; $10^{-9.5}$ at $\lambda = 1.2\mu$; $10^{-8.5}$ at $\lambda = 1.6\mu$;
 $10^{-7.5}$ at $\lambda = 2.0\mu$; $10^{-6.5}$ at $\lambda = 2.4\mu$;

Image Plane PSFs



Planet at tip of flower. Planet intensity 10^{-10} times intensity of star at $\lambda = 0.8\mu$.

Optimization Problem

$$\text{minimize } \int_0^\infty A(r)rdr$$

$$\begin{aligned} \text{subject to } & -10^{-c_j} \leq \Re(E_j(\rho)) \leq 10^{-c_j}, \quad 0 \leq \rho \leq 4, j = 1, 2, \dots, 5 \\ & -10^{-c_j} \leq \Im(E_j(\rho)) \leq 10^{-c_j}, \quad 0 \leq \rho \leq 4, j = 1, 2, \dots, 5 \\ & A(r) = 1, \quad 0 \leq r \leq 6.25 \\ & A'(r) \leq 0, \quad 6.25 \leq r \leq 20 \\ & A(r) = 0, \quad 20 \leq r \leq \infty \\ & -\gamma \leq A''(r) \leq \gamma, \quad 6.25 \leq r \leq 20, \end{aligned}$$

where

- γ is a smoothness parameter,
- c_j specifies the level of contrast at wavelength λ_j ,
- $z = 18,000$ km, and
- by Babinet's principle

$$E_j(\rho) = 1 - \frac{2\pi}{i\lambda_j z} \int_0^\infty e^{\frac{\pi i}{\lambda_j z}(r^2 + \rho^2)} A(r) J_0 \left(\frac{2\pi r \rho}{\lambda_j z} \right) r dr$$